TAKAGI-SUGENO REASONING PROCEDURE FOR PATTERN RECOGNITION

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Abstract. The problem of pattern recognition is well known as well as the reasoning procedure of Takagi-Sugeno (T-S), but up till now both of them have not been used together to describe solutions of the same fuzzy problems. Therefore, the paper analyses the pattern recognition process as Takagi-Sugeno reasoning procedure and defines (T-S) rules weights, solving special linear programming problem, which is constructed according to experts’ fuzzy knowledge. Such an approach for pattern recognition problem has been used for the first time. A practical, comprehensible and usable theory enabling practitioners to build adequate hardware/software tools for pattern recognition is presented in the article.

Keywords: pattern recognition, Takagi-Sugeno reasoning, fuzzy rule, linear programming problem.

1 Introduction

The pattern recognition problem is old enough, very well-known and perfectly described in scientific literature. The work of R. O. Duda and others [1] may be described as one of the highly recommended texts concerning this topic and including an extremely good list of references. In general, a formulation of this problem itself implies a pretty good amount of fuzziness. According to [2], pattern recognition is defined as a search for structure in data, which is performed in three steps: data acquisition, the extraction of pattern features from data by the reduction of dimensionality and the mapping of extracted features to pattern classes. The main problem, related with the issue, is connected with techniques aiming to cope with the problem of fuzziness of expert knowledge.

There are three main goals of this article.
1. Considering the fuzziness of the description of pattern recognition problem in general, to use for the first time Takagi-Sugeno (T-S) fuzzy reasoning procedure, which is known and successfully used in other applications.
2. To propose to apply solutions of specially formulated linear programming problem (LPP) for calculation of weights in (T-S) reasoning procedure instead of using time consuming and poorly determined neural-type training processes. It is considered that formulation of LPP itself, using expert’s knowledge, is more adequate to the fuzzy description of the pattern recognition problem.
3. To present a practical, comprehensible and usable theory enabling practitioners to build adequate hardware/software tools for pattern recognition.

Successful implementations of the proposed theory in different fields will be summarized and presented in the next article of the authors. The inference of ordinary fuzzy systems is based on: 1) a deriving verbal (linguistic) or parametric consequents by pre-processing lists of fuzzy rules that contain verbal or parametric antecedents linked by certain fuzzy logic operations, and 2) a defuzzification process, applying some compositional rule or formulae [2], [3].

The types of rules are:

\[
\text{IF } x \text{ is } A \text{ AND } y \text{ is } B \text{ THEN } z \text{ is } C \quad (\text{for Mamdani fuzzy models})
\]

\[
\text{IF } x \text{ is } A \text{ AND } y \text{ is } B \text{ THEN } z = F(x, y) \quad (\text{for T–S fuzzy models})
\]

Defuzzification procedures for the two cases mentioned above can be described as reasoning on the basis of a set of consequents \( C \) using the centre of gravity (CoG) or a mean of maximum (MoM) methods for Mamdani type models, and a fuzzy mean (FM) method as reasoning by evaluation of all results \( z \) included and processed according to the certain formula \( \Phi(z) \) for T-S models.

2 Pattern Recognition Based on T-S Reasoning Procedure

A pattern of a class is considered (as a physical or abstract structure of class’s objects) described by a set of distinctive features as described in [1] and [2]. The pattern recognition problem can be formulated as follows. It is considered in the article that \( p = 1, 2, \ldots, S \) classes of objects and each class \( p \) has its own pattern. Each object is described by \( n = 1, 2, \ldots, i, j, \ldots, N \) features. After the extraction of a feature as well as the procedures of measurement and normalization as described in [4], the i-th feature of an object that belongs to
the p-th class (i.e. corresponding to the p-th pattern) can be represented by a real number \( \alpha_p \), which expresses a degree of intensity of this particular feature, and the object is represented by a vector-row \( \vec{\alpha}_p = (\alpha_{p_1}, \alpha_{p_2}, \ldots, \alpha_{p_i}, \ldots, \alpha_{p_N}) \). If there are \( l = 1, 2, \ldots, k, \ldots, L \) objects and their dependence to class \( p \) (i.e., they originate from the p-th pattern) is known in advance, class \( p \) is represented by a set of vectors \( \vec{\alpha}_p^l, \forall p, l \).

The main task of the pattern recognition procedure involves the development of T-S rules and defuzzification instruments. Rules and instruments must be defined applying all available information about patterns, which is stored in sets of \( \vec{\alpha}_p^l, \forall p, l \) and experts’ experience that is presented in a verbal form and was collected when working with features of objects and patterns. Such reasoning allows constructing an instrument of pattern recognition providing a possibility to assign any unknown but properly described object \( \vec{x} \) to any of possible classes (and patterns) \( p \). An accuracy of the assignment depends on the instrument’s decision making efficiency to process fuzzy information.

Frequently, better reasoning results are achieved when features of objects are normalized and centred [4], [5]. Then the object is represented as a vector-row \( \vec{\alpha}_p^o = (\alpha_{p_1}^o, \alpha_{p_2}^o, \ldots, \alpha_{p_i}^o, \ldots, \alpha_{p_N}^o) \) the components of which are calculated as follows:

\[
\alpha_{pi}^o = \alpha_{pi} - \frac{1}{N} \sum_{j=1}^{N} \alpha_{pj}^i
\]

Therefore, T-S reasoning procedure can be expressed using Eq (1). It means that a set of positive rules ([6]) for T-S pattern recognition procedure consists of a list of statements:

**IF** < degree of certainty that feature \( i \) with intensity \( x_i \) belongs to the pattern \( p \) is \( K_{pi} > \) \**THEN** \( z_i^+ = K_{pi} x_i^o > \) RECOMMENDED

It means:

\[
IF < \mu^+(x_i^o) = K_{pi} \rightarrow THEN \ z_i^+ = K_{pi} x_i^o, \forall p, i
\]

Similarly, a set of negative rules ([6]) for T-S pattern recognition procedure consists of a list of statements:

**IF** < degree of certainty that feature \( i \) with intensity \( x_i \) belongs to any other pattern except \( p \) is \( 1 - K_{pi} > \) \**THEN** \( z_i^- = 1 - K_{pi} x_i^o > \) NOT RECOMMENDED

This means:

\[
IF < \mu^-(x_i^o) = (1 - K_{pi}) \rightarrow THEN \ z_i^- = -K_{pi} x_i^o, \forall p, i
\]

According to the concept of hyper-inference ([6])

\[
\mu(x_i^o) = \max \{ \mu^+(x_i^o), (1 - \mu^-(x_i^o)) \}
\]

and

\[
z_i = z_i^+ - z_i^- = K_{pi} x_i^o, \forall p, i.
\]

When the unknown object \( \vec{x}^o \) is considered, its degree of belonging to class \( p \) can be evaluated by a dependence function:

\[
\Phi_p (\vec{x}^o) = \sum_{i=1}^{N} x_i^o K_{pi}, \forall p
\]

This defuzzification method for T-S procedure in a vector notation is expressed as:

\[
\Phi_p (\vec{x}^o) = \vec{x}^o K_p^T, \forall p,
\]

where \( T \) stands for a transposition of a certainty vector

\[
\vec{K}_p = (K_{p_1}, K_{p_2}, \ldots, K_{p_i}, \ldots, K_{p_N})
\]

A block diagram, representing a final decision making act of fuzzy pattern recognition based on T-S reasoning procedure, is shown in Figure 1.
In practice, there is no possibility to formulate verbally understandable lists of rules for sets presented by Eq. (3) and (4). Usually neural-type training procedures based on gradient methods are used [1]. An intention to avoid comparatively unpredictable and very clumsy neural-type training procedures for the determination of certainty vectors $\vec{K}_p$, $\forall p$ in T-S reasoning procedure leads to a formulation of an objective function maximization problem, subjected to a set of constrains and constructed according to fuzzy information. In the next section of this paper a special linear programming problem (LPP) is proposed to find optimal certainty vectors $\vec{K}_p$, $\forall p$ for (3)-(6). LPP problem is formulated according to experts' fuzzy information which enables to define rule weights in T-S reasoning based pattern recognition procedure.

3 Linear Programming Problem for Determination of T-S Procedure Rule Weights

A structure of Eq. (7) implies a simple (linear) form of an objective function to be maximized as well as linearity of constraints. According to the description of pattern recognition problem presented in section 2 of this paper, information about the patterns is stored in the sets of $\alpha_p^{oi}$, $\forall i$. In spite of its fuzziness, the formulation of a problem for the determination of certainty vectors $K_{pi}$, $\forall i$ for T-S reasoning procedure can be constructed as follows. One object $\alpha_p^{ok}$, which represents class $p$ and is central, is randomly selected and coordinates of certainty vector $K_{pi}$, $\forall i$ are found in order the dependence function $\Phi_p (\vec{a}_p)$ would be maximized:

$$\Phi_p (\vec{a}_p) = \sum_{i=1}^{N} \alpha_p^{ok} K_{pi} \rightarrow \text{max}$$

under the following constrains:

$$\sum_{i=1}^{N} \alpha_p^{oi} K_{pi} \geq \gamma \sum_{i=1}^{N} \alpha_p^{ok} K_{pi}, \forall l$$

$$\sum_{i=1}^{N} \alpha_p^{oi} K_{pi} \leq \kappa \sum_{i=1}^{N} \alpha_p^{ok} K_{pi}, \forall r \neq p, \forall l.$$  \hspace{1cm} (12)

Eq. (11) is tightly connected with the concept of “positive similarities” within the class and extracts them using the set of positive rules, as it was described in section 2. Eq. (12) reflects dissimilarities between certain classes (in this particular case class $p$) and all other classes (or “negative similarities”) and extracts those dissimilarities by the set of negative rules. Optimal values of $\gamma$ and $\kappa$ are recommended from the interval $[0-1]$, and $\gamma > \kappa$ [4]. Particular values depend on the prior knowledge (or guess) concerning the structure (internal connections and dispersion of patterns’ features) of classes (or patterns). The difference $\gamma - \kappa$ determines experts’ fuzzy assumption concerning a possible structure of classes under investigation and must be chosen on the basis of some experience [4]. Coefficients $\gamma$ and $\kappa$ allow controlling fuzzy level of those similarities and dissimilarities.
As it can be easily noticed the problem belongs to the class of LPP where inequalities (11) and (12) need additional constrains:

\[ 0 \leq K_{pi} \leq A, \forall i, \]  

(13)

where \( A \) is any practically convenient real number.

A solution of Eq. (10)-(13) for the class \( p \) consists of the obtained value of

\[ \max \Phi_p^k(\alpha_p^k) = \Phi_p^{\text{max}}, \]

\[ \tilde{K}_p = (K_{p1}, K_{p2}, \cdots, K_{pi}, \cdots, K_{pN}) \]  

(14)

The procedure must be repeated for all classes \( p \). In this way a set of \( S \) solutions will be obtained and the recognition procedure must be performed according to the Figure 3 taking into account the need of fulfilling a condition of proportionality:

\[ c_1 \Phi_1^{\text{max}} = \cdots = c_p \Phi_p^{\text{max}} = \cdots = c_s \Phi_s^{\text{max}} = B, \]  

(15)

where \( c_i \) are real numbers.

Eq. (15) plays a role of normalization and it means that the fuzzy term very similar must be evaluated by the same number \( B \) whatever pattern is considered.

As it was delivered before, T-S reasoning procedure presents degrees of certainty, indicating that the description of an unknown object \( \bar{x}^o \) belongs to pattern \( p \) (see Figure 1). If there is a need to use this information as a recommendation for a decision maker, the value \( u_p = \Phi_p(\bar{x}^o) / \max_{\forall r} \Phi_r(\bar{x}^o) \) must be determined.

Practically an interval for \( u_p \) is [0-1] with some zone of accuracy as it is shown in the Figure 2, where the degree of certainty \( \mu_p(\bar{x}^o) \) is presented.

![Figure 2. Recommended certainty that \( \bar{x}^o \) belongs to \( p \)](image-url)

When \( u_p \) is in the zone, special additional investigations of the object’s properties are strongly recommended before the final decision is taken.

The entire classification process is shown in Figure 3 and represents the process of pattern recognition combined with decision making recommendations.
4 Final remarks

In the analytical part of this paper a new approach to the problem of fuzzy pattern recognition was presented. First of all the rule-based fuzzy inference, concerning the measure of patterns’ similarity, was introduced, then a description of the process of pattern recognition was based on T-S reasoning procedure, enhanced by specific decision making recommendations. The weights of rules in T-S procedure were defined, solving special LPP, constructed according to the fuzzy information presented or supposed in advance.

A practical, comprehensible and usable theory, enabling practitioners to build adequate hardware/software tools for pattern recognition is presented in the article. Successful implementations of the proposed theory in different fields will be summarized and presented in the next article of the authors of this paper.

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